

# Energy Correlators in Hadron and Nuclear Physics

## Abstract

Energy correlators have re-emerged as a common language for precision QCD, hadronization physics, spin structure, and nuclear dynamics. The standard two-point energy-energy correlator (EEC) contains two distinct and experimentally accessible kinematic regimes. In the back-to-back limit it enters the standard transverse-momentum-dependent (TMD) factorization problem, with a soft function whose rapidity evolution makes the Collins-Soper kernel a natural object of interest. In the small-angle limit the same observable resolves a different hierarchy: a perturbative collinear domain, a transition region, and a post-confinement or free-hadron regime. The near-side regime is now described by a dihadron-fragmentation formulation, a resummed treatment, a global fit over  $Q = 29.0\text{--}91.2$  GeV with a transition scale around 2.3 GeV, and direct hadronization-sensitive measurements in jets at the LHC and RHIC [5,9-19].

The same energy-flow logic also generates a connected family of observables adapted to hadron and nuclear structure. Nucleon energy correlators, semi-inclusive energy correlators, and one-point energy correlators access target- and current-fragmentation dynamics, TMD moments, spin asymmetries, and, in special small- $x$  limits, saturation-sensitive dipole physics [20-33]. These adaptations are related but not identical. The nonperturbative object isolated by a target-fragmentation nucleon correlator differs from that entering a jet-based one-point transversity observable, and both differ from the dipole-amplitude control of the small- $x$  one-point DIS observable.

The standard EEC contains a back-to-back limit and a small-angle limit, while NEC, SIEC, OPEC, and related observables extend the same energy-flow framework to hadron and nuclear structure. The back-to-back limit is governed by TMD factorization and Collins-Soper evolution, whereas the near-side limit probes the transition from perturbative radiation to hadronization.

## 1. Introduction

Energy correlators entered QCD through the classic Basham-Brown-Ellis-Love analyses of  $e^+e^-$  annihilation, where angular energy correlations were proposed as sharp tests of asymptotically free perturbation theory [1,2]. The modern resurgence began from several directions at once. Precision collider theory sharpened the back-to-back factorization problem [4,6]. A second turning point was the conformal-collider and light-ray revival, originating from the Hofman-Maldacena perspective on energy flow and then generalized into the operator language now used for QCD energy correlators [5,15,38]. A third turning point was the realization that the small-angle regime

is not exhausted by perturbative scaling: it contains a transition region and a post-confinement region with its own formal and experimental structure [9–17,37]. Experimental programs at LEP, the LHC, and RHIC then turned these observables into quantitative tools for extracting scales, discriminating hadronization patterns, and testing medium response [16–19,34–36]. In parallel, the same energy-flow logic was carried into nucleon structure, spin physics, and small- $x$  dynamics through nucleon energy correlators (NECs), semi-inclusive energy correlators (SIECs), one-point energy correlators (OPECs), and transverse energy-energy correlators (TEECs) [20–33].

The standard two-point EEC already contains two distinct kinematic problems. In the back-to-back limit, recoil is parametrically small, soft radiation is the leading organizing structure, and the observable becomes a TMD factorization problem. In the small-angle limit, the same EEC instead resolves the transition from perturbative collinear dynamics to a post-confinement or free-hadron regime. These two limits are not two unrelated observables; they are two asymptotic faces of the same operator. The formal descriptions of the two limits are nevertheless different.

The back-to-back limit governs soft-radiation physics, TMD evolution, and precision studies [6–9,18,19], while the small-angle limit governs perturbative scaling, the transition region, and post-confinement dynamics [9–17]. NEC, SIEC, OPEC, spin-sensitive correlators, small- $x$  TEEC/OPEC, and medium extensions realize the same energy-flow framework in hadron and nuclear structure [20–33].

## 2. Standard EEC: definitions, kinematics, and regime structure

The standard EEC is most compactly defined as a two-point energy-flow operator inserted into a state created by a hard source [5,10],

$$\text{EEC} = \frac{\int d^4x e^{iQ \cdot x} \langle 0 | O^\dagger(x) \mathcal{E}(\hat{n}_1) \mathcal{E}(\hat{n}_2) O(0) | 0 \rangle}{\int d^4x e^{iQ \cdot x} \langle 0 | O^\dagger(x) O(0) | 0 \rangle}.$$

Here  $O$  is the hard source that creates the energetic final state, and  $\mathcal{E}(\hat{n}_1)$ ,  $\mathcal{E}(\hat{n}_2)$  measure the energy flux through detectors placed at angles  $\hat{n}_1$  and  $\hat{n}_2$ . The detector operator itself is

$$\mathcal{E}(\hat{n}) = \int_0^\infty dt \lim_{r \rightarrow \infty} r^2 n^i T^{0i}(t, r\hat{n}),$$

written in terms of the stress tensor and the detector direction  $\hat{n}$  [5,25,38]. With this definition, the EEC is a correlation of two energy-flow insertions. It is neither a hadron-tagged observable nor a one-point energy-flow measurement such as OPEC.

The event-level form is the familiar weighted angular distribution

$$\frac{d\Sigma}{d\chi} = \sum_{i,j} \int d\sigma_{ij} \frac{E_i E_j}{Q^2} \delta(\chi - \theta_{ij}).$$

The sum runs over final-state hadrons or partons  $i$  and  $j$ ,  $E_i$  and  $E_j$  are their energies,  $\theta_{ij} = \chi$  is their opening angle, and  $Q$  is the hard scale of the event [10]. In this form the same observable can be pushed to two different edges of phase space. The back-

to-back problem is reached when  $\chi \rightarrow \pi$ , equivalently  $z = (1 - \cos \chi)/2 \rightarrow 1$ . The small-angle problem is reached when  $\chi \rightarrow 0$ , or  $z \rightarrow 0$ . The two edges belong to the same observable, but they are not governed by the same theorem language.

To prepare the back-to-back discussion, it is useful to trade the angular variable for a recoil momentum [6],

$$\frac{d\sigma}{dz} = \frac{1}{2} \sum_{ij} \int dx_i dx_j x_i x_j \frac{d^3\sigma}{dx_i dx_j dz}, \quad \frac{d^3\sigma}{dx_i dx_j dz} = \int d^2\vec{k}_\perp \frac{d^3\sigma}{dx_i dx_j d^2\vec{k}_\perp} \delta\left(1 - z - \frac{\vec{k}_\perp^2}{Q^2}\right).$$

The variables  $x_i$  and  $x_j$  are the energy fractions of the two measured hadrons, while  $\vec{k}_\perp$  is the net recoil transverse momentum that vanishes in the exact back-to-back limit. The distance from  $z = 1$  is therefore measured by  $k_\perp^2/Q^2$ , placing the back-to-back EEC in the same recoil-sensitive class as TMD factorization problems [3,6].

The small-angle side is best separated with a second variable,

$$\zeta = \frac{1 - \cos \chi}{2}, \quad \frac{d\zeta}{d\chi} = \frac{\sin \chi}{2}, \quad \frac{d\Sigma}{d\chi} = \frac{\sin \chi}{2} \frac{d\Sigma}{d\zeta}.$$

For  $\chi \ll 1$ , one has  $\zeta \simeq \chi^2/4$  and  $d\zeta/d\chi \simeq \chi/2$  [15]. If the left-of-peak post-confinement region is approximately flat in  $d\Sigma/d\zeta$ , then the same physics appears approximately linear in  $d\Sigma/d\chi$ . This is a kinematic Jacobian effect, not a dynamical equivalence between the post-confinement and perturbative small-angle domains.

The standard EEC therefore contains a recoil-sensitive back-to-back limit and a small-angle limit with perturbative, transition, and post-confinement regions. The visual appearance of the latter depends on the angular variable, whereas the underlying regime structure does not.

### 3. Back-to-back EEC in the TMD and precision regime

In the back-to-back limit,  $1 - z \sim k_\perp^2/Q^2 \ll 1$ , the differential cross section factorizes at leading power as [6]

$$\frac{d^3\sigma}{dx_i dx_j d^2\vec{k}_\perp} = H(Q, \mu) \int d^2\vec{k}_{\perp,i}^h \int d^2\vec{k}_{\perp,j}^h \int d^2\vec{k}_{\perp,s} \delta^{(2)}\left(\vec{k}_\perp - \left(\frac{\vec{k}_{\perp,i}^h}{x_i} + \frac{\vec{k}_{\perp,j}^h}{x_j} - \vec{k}_{\perp,s}\right)\right) F_{q \rightarrow i}(\vec{k}_{\perp,i}^h, x_i, \mu, \nu) F_{q \rightarrow j}(\vec{k}_{\perp,j}^h, x_j, \mu, \nu) S_{\text{EEC}}(\vec{k}_\perp, \mu, \nu).$$

Here  $x_i$  and  $x_j$  are the energy fractions of the two measured hadrons,  $\vec{k}_{\perp,i}^h$  and  $\vec{k}_{\perp,j}^h$  are their transverse momenta with respect to the two nearly lightlike jet directions  $n$  and  $\bar{n}$ ,  $\vec{k}_{\perp,s}$  is the recoil carried by soft radiation, and  $\vec{k}_\perp$  is the total imbalance conjugate to  $1 - z$ . The scales  $\mu$  and  $\nu$  are the ultraviolet and rapidity renormalization scales, respectively. The hard function  $H(Q, \mu)$  controls the short-distance production, the TMD fragmentation functions  $F_{q \rightarrow i}$  and  $F_{q \rightarrow j}$  encode the two collinear jets, and the recoil-sensitive soft function  $S_{\text{EEC}}$  supplies the rapidity-sensitive broadening. This factorization formula identifies the recoil-sensitive TMD dynamics probed by the back-to-back EEC.

The corresponding soft sector has the operator form [6]

$$y_\nu(\vec{b}_\perp) = \left( \frac{ib_0}{\nu}, \frac{ib_0}{\nu}, \vec{b}_\perp \right), \quad b_0 = 2e^{-\gamma_E},$$

$$S_{\text{EEC}}(\vec{b}_\perp, \mu, \nu) = \lim_{\nu \rightarrow +\infty} \frac{1}{N_c} \text{tr} \langle 0 | T \left[ S_{\bar{n}+}^\dagger(0) S_{n-}(0) \right] \bar{T} \left[ S_{n+}^\dagger(y_\nu(\vec{b}_\perp)) S_{\bar{n}-}(y_\nu(\vec{b}_\perp)) \right] | 0 \rangle.$$

The vectors  $n$  and  $\bar{n}$  are the two lightlike jet directions,  $\vec{b}_\perp$  is the transverse impact parameter conjugate to recoil momentum, and  $b_0 = 2e^{-\gamma_E}$  is the standard Fourier-transform constant. The objects  $S_{n\pm}$  and  $S_{\bar{n}\pm}$  are eikonal Wilson lines,  $T$  and  $\bar{T}$  denote time and anti-time ordering,  $N_c$  is the number of colors, and  $y_\nu(\vec{b}_\perp)$  is the rapidity-regulated transverse shift used to define the soft matrix element. This formula identifies the soft object whose rapidity evolution feeds the Collins–Soper-sensitive structure.

The relation to the SIDIS/TMD soft sector is [24]

$$S_{n\bar{n}}(b_\perp, \mu, \nu) = S_{\text{EEC}}\left(b_\perp, L_\nu + \ln \frac{n \cdot \bar{n}}{2}, \mu\right).$$

Here  $S_{n\bar{n}}$  is the standard DIS/TMD soft function,  $b_\perp = |\vec{b}_\perp|$ , and  $L_\nu$  denotes the regulator-dependent rapidity logarithm. The same rapidity-sensitive soft physics therefore enters both the back-to-back EEC and DIS TMD factorization. The Collins–Soper kernel follows from the rapidity evolution of this common soft sector [5,7,8,24].

After the energy weight over hadrons is taken, the EEC jet function obeys the sum-rule relation [24]

$$J_f(b_\perp, E_{\bar{n}}, \mu, \nu) \equiv \sum_h \int_0^1 dz z F_{h/f}^{\text{OPE}}\left(z, \frac{b_\perp}{z}, E_{\bar{n}}, \mu, \nu\right) = \sum_i \int_0^1 d\xi \xi C_{if}\left(\xi, \frac{b_\perp}{\xi}, E_{\bar{n}}, \mu, \nu\right),$$

where  $J_f$  is the flavor- $f$  EEC jet function,  $E_{\bar{n}}$  is the energy flowing into the  $\bar{n}$ -collinear jet,  $F_{h/f}^{\text{OPE}}$  is the OPE-matched TMD fragmentation function into hadron  $h$ , and  $C_{if}$  are the perturbative matching coefficients after the energy-weighted sum rule is applied. The variables  $z$  and  $\xi$  are fragmentation fractions. The weighted final state is thus expressed in terms of the same matching coefficients that appear in TMD fragmentation, and the back-to-back EEC corresponds to a definite projection of the TMD fragmentation problem.

This operator structure also underlies the precision discussion. The analysis of Ref. [7] treats the observable as simultaneously sensitive to  $\alpha_s$  and the Collins–Soper kernel, and accordingly extracts  $\alpha_s(m_Z)$  together with a nonperturbative component of rapidity evolution. Ref. [8] instead emphasizes the limited constraining power of existing data for the kernel and questions the robustness of the quoted extraction. The difference between the two analyses lies in the level of sensitivity attributed to current measurements, not in the factorization theorem itself.

The experimental situation is already sufficiently developed for this question to be quantitative. The CMS jet-energy-correlator analysis at  $\sqrt{s} = 13$  TeV with  $36.3 \text{ fb}^{-1}$

extracted

$$\alpha_S(m_Z) = 0.1229_{-0.0050}^{+0.0040},$$

using a ratio of two- and three-point correlators inside jets [18]. This extraction is distinct from the inclusive  $e^+e^-$  EEC, since the CMS measurement is based on a jet correlator rather than the canonical event-shape observable.

**Figure 1.** *Back-to-back precision line.* Comparison of the CMS correlator-ratio extraction [18] with the archived ALEPH back-to-back EEC analysis at 91.2 GeV [19], illustrating precision applications of the same observable family in hadron-collider and  $e^+e^-$  environments.

The corresponding  $e^+e^-$  comparison is provided by the archived ALEPH  $Z$ -pole analysis at 91.2 GeV, which brings the back-to-back EEC into direct contact with modern source-unfolded data in the original environment of the observable [19]. In parallel, Ref. [9] shows that nonperturbative effects enter numerically once the analysis is extended beyond a purely asymptotic description [9].

The back-to-back factorization theorem is by now well established, and its relation to TMD rapidity evolution is explicit. In this regime, the Collins–Soper kernel and  $\alpha_s$  extraction are tied to the same rapidity-sensitive soft dynamics.

Polarized and recoil-sensitive extensions show that this logic is not confined to the unpolarized  $e^+e^-$  case. In spin-dependent DIS, the energy-weighted correlator in the current-fragmentation, nearly back-to-back region takes the form [24]

$$y \frac{d\sigma_{\ell+N \rightarrow \ell'+X}^{\text{EC}}}{d^2\vec{q}_\perp dx dy} \simeq \sum_f \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \cdot \vec{q}_\perp} [H_f(Q^2, \mu) S_{n\bar{n}}(b_\perp, \mu, \nu)] J_f(b_\perp, E_{\bar{n}}, \mu, \nu) [\sigma_0^U x B_{f/N}(x, b_\perp, E_n, \mu, \nu) + \lambda_\ell S_\parallel \sigma_0^L]$$

Here the sum runs over parton flavors  $f$ ;  $x$  and  $y$  are the usual DIS Bjorken and inelasticity variables,  $\vec{q}_\perp$  is the measured transverse recoil and  $\vec{b}_\perp$  its Fourier-conjugate variable,  $E_n$  and  $E_{\bar{n}}$  are the characteristic energies along the beam and current directions,  $\lambda_\ell$  is the lepton helicity, and  $S_\parallel$  is the longitudinal target polarization. The beam functions  $B_{f/N}$  and  $\Delta B_{f/N}$  are the unpolarized and polarized TMD beam functions, while  $\sigma_0^{U,L}$  are the leptonic prefactors multiplying them. The back-to-back EC in DIS is governed by the same EEC jet function and the same soft sector encountered in the standard problem, while the spin dependence enters through the polarized beam function  $\Delta B_{f/N}$ . The same recoil-sensitive factorization structure thus extends into spin-dependent DIS [24].

Collins-type energy correlators and azimuthally dependent correlators were proposed as probes of nucleon structure and TMD dynamics in SIDIS-like settings [28,29]. Ref. [24] further shows that the same observable family resolves proton spin differently in the current-fragmentation and target-fragmentation limits, with the former tied to the back-to-back TMD theorem and the latter to a nucleon-correlator description. The back-to-back spin extension therefore remains within the recoil-sensitive standard-EEC framework, whereas the target-fragmentation observables belong to the broader adaptation family.

The back-to-back regime provides direct access to TMD rapidity evolution, quanti-

tative phenomenology, and polarized extensions within the same factorized structure.

#### 4. Small-angle EEC from perturbative scaling to post-confinement dynamics

The small-angle limit contains several distinct layers: perturbative scaling, the transition region and its extracted scales, the dihadron-fragmentation and resummation descriptions of the near-side distribution, and the light-ray-OPE interpretation of post-confinement scaling [5,9-17,37]. The transition peak separates two small-angle domains. To the right of the peak, the distribution remains perturbative and resummation-controlled. To the left, it enters a post-confinement or free-hadron regime whose appearance depends on the chosen variable, while the underlying physics does not.

This distinction is visible already in the Jacobian relation introduced in Section 2,

$$\zeta = \frac{1 - \cos \chi}{2}, \quad \frac{d\Sigma}{d\chi} = \frac{\sin \chi}{2} \frac{d\Sigma}{d\zeta}.$$

A plateau-like behavior in  $d\Sigma/d\zeta$  therefore becomes approximately linear in  $d\Sigma/d\chi$  at small  $\chi$ . The variable choice changes the visual slope, but it does not convert the post-confinement side into the perturbative scaling law that governs the right side of the peak [10,15].

This separation is now quantitatively visible in data. The ALICE in-jet EEC measurement in  $pp$  collisions at  $\sqrt{s} = 5.02$  TeV showed that the distributions in different jet- $p_T$  bins collapse onto a common scaled peak at

$$\langle p_T^{\text{chjet}} \rangle_{R_L} = 2.39 \pm 0.17 \text{ GeV}/c$$

when plotted against  $\langle p_T^{\text{chjet}} \rangle_{R_L}$  [16]. The perturbative side is anchored by a normalization window  $[12 \text{ GeV}/c / \langle p_T^{\text{chjet}} \rangle, 0.4]$ , while the free-hadron side is fit in the window  $[0.01, 0.7 \text{ GeV}/c / \langle p_T^{\text{chjet}} \rangle]$  [16]. The peak position scales with the jet momentum and isolates a common hadronization-related transition scale.

The STAR measurement at RHIC exhibits the same structure at lower energy and with explicit charge sensitivity [17]. Using the fit parameter  $\langle p_{T,\text{jet}} \rangle^2 T$ , the three jet bins correspond to transition peaks around 2.85, 2.79, and 2.74 GeV/c [17]. In the charge-selected analysis, the ratio (Like – Opposite)/Inclusive reaches roughly  $-0.30$  near  $R_L \sim 0.1$  and then rises back toward zero as  $R_L \rightarrow 1$  [17]. The charge dependence is therefore concentrated near the transition and hadronization region, where current Monte Carlo descriptions remain quantitatively insufficient.

**Figure 2.** *Near-side transition scales in jets.* Comparison of the scaled EEC measurements from ALICE and STAR [16,17], showing that the extracted transition scale remains clustered around 2.3-2.9 GeV/c across collider energies and jet- $p_T$  bins.

**Figure 3.** *Charge sensitivity in the hadronization region.* STAR charge-selected EEC distributions [17], showing that the strongest charge dependence appears near

the transition and hadronization region rather than in the asymptotic perturbative tail.

These measurements are consistent with the theoretical picture developed in Refs. [11,12], where the left-of-peak region is treated as a distinct regime with nonperturbative information that can be compared across processes. In this region, the DiFF and light-ray-OPE descriptions are complementary.

A theorem-level near-side description now exists in terms of dihadron fragmentation. The kinematic bridge is [13]

$$z_{12} = \frac{R_T^2}{Q^2} \frac{\tau^2}{\tau_1^2 \tau_2^2},$$

with  $\tau_{1,2} = 2E_{1,2}/Q$  the energy fractions of the two hadrons in the pair,  $\tau = \tau_1 + \tau_2$ , and  $R_T$  their relative transverse momentum. This equation converts the hadron-pair relative transverse scale into the standard near-side EEC variable. The associated nonperturbative object is the EEC-weighted dihadron fragmentation function,

$$\mathcal{D}^i(z_\chi, Q^2) = \sum_{h_1, h_2} \int d\xi_1 d\xi_2 d^2\vec{R}_T \delta\left(z_\chi - \frac{R_T^2}{Q^2} \frac{\xi^2}{\xi_1^2 \xi_2^2}\right) \xi_1 \xi_2 D_1^{h_1 h_2 / i}(\xi_1, \xi_2, \vec{R}_T),$$

where  $\xi_{1,2}$  are the momentum fractions carried by the two hadrons,  $\xi = \xi_1 + \xi_2$ , and  $D_1^{h_1 h_2 / i}$  is the ordinary unpolarized dihadron fragmentation function of parton  $i$ . The derived object  $\mathcal{D}^i$  is the EEC-DiFF relevant for the free-hadron and transition region [13].

In the DiFF description, the EEC is written as

$$\text{EEC}(\chi) \Big|_{\text{DiFF}} = \frac{\sin \chi}{2} \frac{\sigma_0}{\sigma_t} \int_0^1 dw w^2 \vec{\mathcal{D}}(z_\chi, w^2 Q^2; \mu) \cdot \vec{H}\left(w; \frac{Q}{\mu}\right).$$

Here  $\sigma_0$  and  $\sigma_t$  are the Born and total cross sections,  $w$  is the partonic energy fraction entering the hard coefficient vector  $\vec{H}$ , and  $z_\chi$  is the near-side angular variable built from  $\chi$ . This is a DiFF-region theorem rather than the full-range singular-plus-nonsingular matched expression. The perturbative large- $R_T$  tail of  $\mathcal{D}^i$  matches onto the usual EEC jet function, so the DiFF and perturbative descriptions are continuously connected without changing observable identity [13].

In a jet-based derivation of the small-angle problem, the unintegrated correlator factorizes as [14]

$$\frac{d\langle \text{EEC}(\theta) \rangle(E_J)}{d^2\theta} = E_J^2 \sum_i \int dx x^4 H_i(x, Q/\mu) \Gamma_i(\mu, q_\perp), \quad q_\perp = x E_J \theta,$$

with the Fourier-space evolution

$$\tilde{\Gamma}_i(\mu, b_T) = \Gamma_j(\mu_b) P \exp \left[ - \int_{\mu_b}^{\mu} d \ln \mu'^2 \gamma(\mu') \right]_{ji}, \quad \mu_b = \frac{2e^{-\gamma_E}}{b_T}.$$

Here  $E_J$  is the jet energy,  $x$  is the energy fraction carried by the initiating parton,  $H_i$  is the hard coefficient for parton flavor  $i$ ,  $\Gamma_i$  is the near-side jet correlator in transverse-momentum space, and  $b_T$  is its conjugate impact parameter. The ordered exponential evolves the correlator with the anomalous-dimension matrix  $\gamma$ , starting from the canonical scale  $\mu_b$ . The near-side problem is thus written as a TMD-like evolution problem in transverse position space, with a jet-function object built from dihadron fragmentation rather than the soft-plus-beam structure of the back-to-back limit [14].

A unified global-fit formulation can be written as [10],

$$\frac{1}{\hat{\sigma}_0} \frac{d\Sigma^{\text{res}}}{dz} = \frac{Q^2}{2} \int_0^\infty db b J_0(bq_T) \int_0^1 dx x^2 \tilde{\mathbf{J}}\left(\frac{b}{x}, \mu\right) \cdot \mathbf{H}(x, Q, \mu), \quad q_T = \sqrt{z} Q = Q \sin(\chi/2),$$

together with the evolution

$$\frac{d}{d \ln \mu^2} \mathbf{H}(x, Q, \mu) = - \int_x^1 \frac{dy}{y} \mathbf{P}(y, \mu) \cdot \mathbf{H}\left(\frac{x}{y}, Q, \mu\right), \quad \frac{d}{d \ln \mu^2} \tilde{j}^i(b, \mu) = \int_0^1 dy y^2 \tilde{j}^i\left(\frac{b}{y}, \mu\right) \cdot \mathbf{P}(y, \mu),$$

supplemented by a fitted nonperturbative profile  $\tilde{j}_{\text{np}}(b) = \exp[-(a_1 b)^{a_2}]$  [10]. Here  $\hat{\sigma}_0$  is the Born-level normalization,  $J_0$  is the Bessel function from the Fourier transform,  $\tilde{\mathbf{J}}$  is the near-side jet function in impact-parameter space,  $\mathbf{H}$  is the hard vector, and  $\mathbf{P}$  is the DGLAP splitting kernel acting in flavor space. The parameters  $a_1$  and  $a_2$  control the fitted nonperturbative profile. In practice this framework describes data over

$$0^\circ < \chi < 90^\circ, \quad Q = 29.0\text{--}91.2 \text{ GeV},$$

and extracts a transition scale around 2.3 GeV with  $\chi^2/N_{\text{pts}} = 86.4/46 = 1.88$  [10,13].

**Figure 4.** *Global near-side fit in  $e^+e^-$ .* Global comparison of the fitted framework with OPAL, SLD, TOPAZ, TASSO, and MARK II data [10], showing a continuous description from the perturbative side through the transition region into the post-confinement side.

The hadronic small-angle OPE can be written as [15]

$$\mathcal{E}(n_1)\mathcal{E}(n_2) = \sum_{J_L, k} A_{J_L, k} C_{J_L}(n_1, n_2, \partial_{n_2}) \mathcal{O}_{J_L, k}^H(n_2), \quad C_{J_L}(n_1, n_2, \partial_{n_2}) = |n_{12}|^{-6-J_L} (1 + \mathcal{O}(|n_{12}|)).$$

The indices  $J_L$  and  $k$  label the hadronic light-ray tower,  $A_{J_L, k}$  are the corresponding Wilson coefficients,  $\mathcal{O}_{J_L, k}^H$  are hadronic light-ray operators, and  $n_{12} = n_1 - n_2$  measures the small angular separation. The near-side limit is described by hadronic light-ray operators of definite boost weight. Ref. [15] shows that the nonperturbative light-ray-OPE coefficients are moments of the dihadron fragmentation function. In the post-confinement and transition region, the DiFF/resummation language and the light-ray-OPE language describe the same near-side hadronization content, in factorized and operator forms, respectively.

The corresponding scaling relation can be written as [15]

$$U_{J_L}^{\text{LL}}(\mu_1^2, \mu_2^2) = \left[ \frac{\alpha_s(\mu_2^2)}{\alpha_s(\mu_1^2)} \right]^{(\gamma_b^{(0)} - J_L - 1)/\beta_0}.$$

The evolution kernel  $U_{J_L}^{\text{LL}}$  evolves a fixed light-ray block between the two hard scales  $\mu_1$  and  $\mu_2$ ;  $\gamma_b^{(0)}$  is the one-loop anomalous-dimension coefficient controlling the block, and  $\beta_0$  is the one-loop QCD beta-function coefficient. In the post-confinement region the dominant block is the  $J_L = 5$  contribution, so the quantum evolution of the plateau region is governed by the  $J = 5$  DGLAP anomalous dimension [15]. The left-of-peak region therefore carries calculable  $Q$ -dependence and enters the confinement-transition analysis of Ref. [9].

The near-side limit thus contains a perturbative right-of-peak region described by collinear factorization and resummation, a transition region constrained by data and fitted scales, and a left-of-peak post-confinement regime whose apparent plateau or linear behavior follows from the plotting variable through the Jacobian. Its scaling behavior is tied to DiFF moments and, in leading quantum evolution, to the  $J = 5$  DGLAP moment. This picture is supported by hadron-collider data,  $e^+e^-$  global fits, dihadron-fragmentation factorization, and light-ray-OPE scaling.

## 5. Energy correlators for hadron and nuclear structure

NEEC, SIEC, OPEC, and related observables are most usefully classified by two distinctions. The first is one-point versus two-point energy flow, so that OPEC is not EEC. The second is target versus current fragmentation, so that a target-fragmentation nucleon correlator is not the same object as a current-fragmentation semi-inclusive correlator even when both can be related to TMD moments. These distinctions determine which nonperturbative structure is isolated in each case [20–33].

### 5.1 5.1 NEEC, SIEC, and OPEC across fragmentation regions

The nucleon energy correlator introduced in Ref. [20] embedded the energy-flow measurement in DIS and identified the target-fragmentation region as a clean observable channel, later connected to semi-inclusive and crossed  $e^+e^-$  constructions [21–24].

The weighted DIS energy correlator interpolates between the target-fragmentation region (TFR) and the current-fragmentation region (CFR) in a common Breit-frame setup [21]. The underlying measurements are weighted cross sections of the form [22]

$$\Sigma_n = \frac{1}{\sigma} \int d\sigma \frac{E(\Omega_1)}{Q} \dots \frac{E(\Omega_n)}{Q} w_n(\Omega_i), \quad \Sigma_n^{(h)} = \frac{1}{\sigma_h} \int d\sigma_h \frac{E(\Omega_1)}{Q} \dots \frac{E(\Omega_n)}{Q} w_n(\Omega_i).$$

Here  $w_n(\Omega_i)$  is the angular weighting function,  $\Sigma_n$  is the target-fragmentation observable normalized to the inclusive DIS cross section  $\sigma$ , and  $\Sigma_n^{(h)}$  is the fragmentation-side

observable normalized to the semi-inclusive cross section  $\sigma_h$  for producing a tagged hadron  $h$ . In the first case the angles  $\Omega_i = (\theta_i, \phi_i)$  are measured with respect to the beam remnant; in the second they are measured around the identified hadron. These definitions make the family relation concrete. The connection is one of observable-family continuity rather than numerical identity.

The semi-inclusive extension sharpened this continuity by showing that SIEC contains NEEC as a special case while also extending the measurement into the current or jet fragmentation region [22]. The cleanest statement from the later crossed analysis is that the collinear limit of semi-inclusive energy correlators in single-hadron  $e^+e^-$  production is the crossed counterpart of the target-fragmentation SIEC/NEEC logic in SIDIS [23]. This relation closes the family at the level of factorization class rather than by analogy alone.

In OPEC-type constructions one inserts a single  $\mathcal{E}(\hat{n})$  rather than the two-point product  $\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)$  [25]. OPEC is therefore an angular energy-flux measurement rather than a correlated pair measurement. OPEC, NEEC, and SIEC are built from the same energy-flow operator, but they do not isolate the same nonperturbative object.

## 5.2 5.2 TMD moments, spin structure, and polarization observables

The NEEC/SIEC program relates energy-flow measurements to TMD-sensitive moments and spin observables [22]. For target fragmentation one defines the total transverse momentum through the energy flow as

$$\vec{k}_t = - \int d\theta d\phi \sin\theta \vec{n}_t \mathcal{E}(\Omega), \quad \vec{n}_t = (\cos\phi, \sin\phi),$$

so that the transverse momentum of the radiation can be written directly in terms of the calorimetric energy flow. In this language the  $n$ -th transverse moment of a TMD PDF is generated by the target-side energy correlator itself.

Defining

$$M_\alpha^{(n)}(x) = \int d^2\vec{k}_t (-k_{t,\alpha})^n q(x, \vec{k}_t),$$

with  $q(x, \vec{k}_t)$  the quark TMD PDF,  $k_{t,\alpha}$  the  $\alpha$ -component of the transverse momentum, and  $x$  the Bjorken momentum fraction, Ref. [22] shows that

$$M_\alpha^{(n)} = \int d\Omega_1 \cdots d\Omega_n n_{1,t,\alpha} \cdots n_{n,t,\alpha} \int \frac{dy^-}{2\pi} e^{-ixy^- P^+} \langle P | \bar{\psi}(y^-, 0) \mathcal{E}(\Omega_1) \cdots \mathcal{E}(\Omega_n) \frac{\Gamma}{2} \psi(0) | P \rangle,$$

which is the moment-generating form of the target-side  $n$ -point energy correlator. Here  $P^+$  is the large light-cone momentum of the incoming hadron,  $y^-$  is the light-cone separation, and  $\Gamma$  is the Dirac projector selecting the relevant quark channel. Choosing the angular weight so that it projects the product of transverse unit vectors

$n_{1,t,\alpha} \cdots n_{n,t,\alpha}$ , one obtains the compact relation

$$M_\alpha^{(n)}(x) = \int w_n(\Omega_i) f_{q,n}(x, \{\Omega_i\}),$$

where  $f_{q,n}$  is the target-side  $n$ -point quark correlator [22]. Target-fragmentation energy correlators thus generate moments of TMD PDFs without imposing back-to-back kinematics.

The fragmentation-side relation is parallel but not identical. If  $d_{i,n}(z, \{\Omega_i\})$  denotes the fragmenting energy correlator for a hadron carrying longitudinal fraction  $z$ , then [22]

$$\int w_n(\Omega_i) d_{i,n}(z, \{\Omega_i\}) = z^2 \int d^2\vec{k}_t (k_{t,\alpha})^n d(z, z\vec{k}_t),$$

with the same angular weight  $w_n$  as above, except that the angles are measured from the observed hadron rather than the beam. The function  $d(z, z\vec{k}_t)$  is the TMD fragmentation function, and  $d_{i,n}$  is the corresponding fragmentation-side energy correlator for parton flavor  $i$ . This equation is the fragmentation-side counterpart of the target-side moment relation, and together the two formulas state the main point of Ref. [22]: NEEC and SIEC generate moments of TMD PDFs and TMD FFs separately. That separation is the advantage over conventional processes in which two unknown TMD objects enter in convolution.

The same framework also isolates specific spin moments. For a transversely polarized target one may choose the azimuthal weight

$$w^{f_1^\perp}(\Omega) = \frac{d\Omega}{\pi} \epsilon_t^{\mu\nu} n_{t,\mu} s_{t,\nu},$$

where  $s_t$  is the transverse spin vector,  $\epsilon_t^{\mu\nu}$  is the antisymmetric tensor in the transverse plane,  $M$  is the hadron mass, and  $\vec{e}_z$  is the unit vector normal to the transverse plane. The resulting one-point target correlator satisfies [22]

$$\int w^{f_1^\perp}(\Omega) f_{q,1}(x, \Omega) = \int \frac{d^2\vec{k}_t}{\pi M} \frac{|\vec{k}_t \times \vec{s}_t \cdot \vec{e}_z|}{2} f_{1T}^\perp(x, \vec{k}_t) = \int \frac{dk_t^2}{2M} k_t^2 f_{1T}^\perp(x, k_t).$$

A suitable angular weight on the target-side energy flow therefore measures the first moment of the Sivers function directly, rather than only through a convolution with a second nonperturbative object. Fragmentation-side weights isolate Collins-sensitive moments in the same spirit [22,26].

A complementary route is provided by the one-point transversity program. At the observable level one measures the asymmetry

$$A_{UT}^{\sin(\phi_s - \phi_n)} = \frac{Z_{UT}}{Z_{UU}}.$$

Here  $\phi_s$  and  $\phi_n$  are the azimuthal angles of the transverse spin and the measured energy-flow direction, while  $Z_{UT}$  and  $Z_{UU}$  are the spin-dependent and spin-averaged structure functions of the one-point correlator [25]. As an angular and infrared-safe

observable, it probes a broader angular range than traditional hadron- $j_{\perp}$  measurements while retaining sensitivity to the same Collins-sector physics [25]. The Collins function nevertheless remains the dominant source of uncertainty.

In the OPEC transversity study, the projected detector angular reach extends down to  $10^{-4}$  rad [25]. From the published curves, the  $\pi^{-}$  single-spin asymmetry reaches roughly  $-0.06$  near  $\theta_n \sim 2 \times 10^{-2}$  at  $\sqrt{s} = 510$  GeV and  $p_T \approx 32.3$  GeV, while the  $\pi^{+}$  channel peaks around  $+0.02$ - $+0.03$  [25]. At  $p_T \approx 50$  GeV the JAM3D-22 fit gives approximately  $+0.035$  for  $\pi^{+}$  and  $-0.075$  for  $\pi^{-}$ , substantially larger in magnitude than the corresponding TMD-evolution curves [25]. The predicted asymmetries are therefore sign-definite and angle resolved.

**Figure 5.** *Spin-sensitive energy flow in jets.* OPEC transversity asymmetries as functions of jet  $p_T$  and  $\theta_n$  [25], showing sizeable, sign-definite angular structure with uncertainties still dominated by the Collins sector.

The relation between this one-point transversity program and the NEEC/SIEC moment program is likewise specific. OPEC provides a one-point angular-energy measurement whose spin asymmetry is sensitive to the same broad transverse structure that NEEC/SIEC access through weighted multipoint correlators. The observables are related at the level of transverse dynamics, but they differ in operator definition, fragmentation support, and factorization formula.

Other spin-sensitive correlators fill out the same picture. Collins-type EECs and azimuthally dependent ECs were proposed as probes of nucleon structure and TMD dynamics [28,29]. The Collins-type fragmentation energy correlator in SIDIS gives a more direct hadron-level access to fragmentation asymmetries in an energy-flow language [26]. Gluon polarimetry through energy correlators introduces a distinct  $\cos 2\phi$  modulation and extends the logic to linearly polarized gluons in jets [27]. These observables are related because they translate spin information into energy-flow correlations; they are not identical because the underlying operator content and kinematic limits differ.

### 5.3 5.3 Small- $x$ dynamics, odderon sensitivity, and nuclear matter

In the small- $x$  sector, the transverse EEC for hadron production turns the spin dependence into a direct odderon-sensitive observable [30]. In the back-to-back region the predicted asymmetry is of order 0.1%, but it increases away from the strict  $\tau \ll 1$  limit. At  $\tau \sim 10^{-1}$ , the plotted  $h^{+}$  asymmetry is about 0.23%, 0.15%, and 0.09% for  $x = 0.010$ , 0.005, and 0.002, respectively [30]. The decrease with smaller  $x$  is consistent with BK evolution suppressing the odderon contribution.

**Figure 6.** *Small- $x$  spin sensitivity.* TEEC Sivvers asymmetry curves at small  $x$  [30], showing a nonzero,  $x$ -dependent signal associated with odderon dynamics rather than generic TMD broadening.

The one-point DIS observable at small  $x$  is cleaner in a more specific sense [31]. After the momentum sum rule is used, the fragmentation dependence cancels and the dipole amplitude remains as the only nonperturbative input. This property is spe-

cific to that branch and does not extend to jet-based OPEC transversity or to generic NEEC/SIEC observables, where fragmentation and moment structure remain essential [25,31].

The small- $x$  and nuclear sectors also admit genuine hadronic- and medium-matter extensions. NECs have been formulated in the Color Glass Condensate, where the correlator probes multi-gluon fields of the target [32]. Odderon-sensitive NECs sharpen that logic in transverse-spin channels [33]. In hadronic and nuclear collisions, jet-energy correlators have been proposed and measured as medium-sensitive probes, with recent work showing how they resolve quark-gluon-plasma scales and how the first in-medium measurements should be interpreted [34–36].

NEEC, SIEC, and their crossed versions recast target- and current-fragmentation physics in energy-flow language. OPEC-type observables turn angular energy flow into spin-sensitive jet probes. Small- $x$  TEEC and OPEC isolate odderon and dipole dynamics in limits where the observable simplifies. Medium and nuclear extensions carry the same framework into  $pp$ ,  $pA$ , and  $AA$  collisions. The relevant nonperturbative object depends on the fragmentation region and on the point multiplicity of the correlator.

## 6. Conclusions and outlook

Energy correlators define a field structured by two limits of the standard EEC and a broader set of related observables. In the back-to-back limit, the EEC is a recoil-sensitive TMD observable with a direct relation to Collins–Soper evolution [6,24]. This regime supports precision phenomenology, explicit fits, and an active discussion of how strongly present data constrain the Collins–Soper kernel [7,8,18,19]. In the small-angle limit, the same EEC resolves a perturbative collinear domain, a transition regime, and a post-confinement or free-hadron domain. This regime now has transition data, evidence for a universal scale, a dihadron-fragmentation framework, a resummed global fit, and a light-ray-OPE interpretation in which the post-confinement quantum evolution is governed by the  $J = 5$  DGLAP moment [9–17].

The two regimes remain tied together by observable identity but are described by different theorem languages. Back-to-back EEC and near-side EEC are not the same factorization problem. Likewise, the left-of-peak post-confinement region is distinct from the right-of-peak perturbative small-angle region. A plateau in one variable and a line in another are related by the Jacobian, not by a change of hadronic regime [10,15].

The same separation applies to the hadron- and nuclear-structure observables. NEC, SIEC, OPEC, TEEC, and their spin-sensitive or small- $x$  variants are related because they translate energy flow into structure information, but they are not interchangeable. They live in different fragmentation regions, use different point multiplicities, and isolate different nonperturbative objects [20–33]. The dipole control of the small- $x$  DIS OPEC is specific to that branch [31], the broad angular reach of OPEC transversity is specific to its jet-based one-point construction [25], and the TMD-moment relations of NEC/SIEC are specific to their weighted semi-inclusive structure

[21-24].

Several open problems remain. On the precision side, the back-to-back regime requires new data and a clearer separation between genuine Collins-Soper sensitivity and parameter degeneracy in present fits [7,8]. On the formal side, the near-side regime already has DiFF and resummation descriptions together with a light-ray-OPE interpretation, but the full relation between these languages is still being developed [13-15]. On the phenomenological side, the most useful hadron- and nuclear-structure observables are those that preserve a sharp observable definition while isolating genuinely new nonperturbative information.

The available data already separate regimes, scales, and competing interpretations, while the observable family connects TMD physics, proton spin, small- $x$ , and nuclear matter without obscuring the role of the standard EEC in its back-to-back and near-side limits.

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